[2] $C(3,7,4)$, $r = 5$

EQN CIRCLE $(x-3)^2 + (y-7)^2 + (z-4)^2 = 25$

At intersection of xy-plane, $z = 0$.

$$(x-3)^2 + (y-7)^2 + (-4)^2 = 25$$

$$x^2 - 6x + y^2 - 14y + 9 + 49 + 16 = 25$$

$$x^2 - 6x + y^2 - 14y = -49$$

$$(x^2 - 6x + 9) + (y^2 - 14y + 49) = -49 + 49 + 9$$

$$(x-3)^2 + (y-7)^2 = 9$$

$\therefore$ circle in xy-plane $C(3,7,0)$, $r = 3$.

At intersection of yz-plane, $x = 0$

$$(-3)^2 + (y-7)^2 + (z-4)^2 = 25$$

$$y^2 - 14y + z^2 - 8z + 9 + 49 + 16 = 25$$

$$y^2 - 14y + z^2 - 8z = -49$$

$$(y-7)^2 + (z-4)^2 = -49 + 49 + 16$$

$\therefore$ Circle in yz-plane $C(0,7,4)$, $r = 4$

[3.1] $A(2,3,-4)$, $B(3,1,-1)$, $C(m, 7, n-1)$

colinear iff $\vec{AB} = t\,\vec{BC}$

$$\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = t \begin{bmatrix} m-3 \\ 7-1 \\ n \end{bmatrix}$$

since $-2 = 6t$, $t = -\frac{1}{3}$.

Then $1 = -\frac{1}{3}(m-3)$

$-3 = m-3$

$\boxed{m = 0}$

$3 = tn$

$3 = -\frac{1}{3} n$

$\boxed{n = -9}$

$\therefore\ m = 0,\ n = -9$

[3.2] <u>SOLUTION 1</u>

$P(4,-2,5)$, $Q(-3,4,-4)$, $R(1,2,4)$, $S(m,1-m,4)$.

P, Q, R in Plane, so

$$\begin{aligned} 4x - 2y + 5z + d &= 0 \\ -3x + 4y - 4z + d &= 0 \\ x + 2y + 4z + d &= 0 \end{aligned} \equiv \begin{bmatrix} 4 & -2 & 5 & 1 & 0 \\ -3 & 4 & -4 & 1 & 0 \\ 1 & 2 & 4 & 1 & 0 \end{bmatrix}$$

$$\equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & 0 \end{bmatrix} \equiv \begin{bmatrix} a = -d \\ b = \frac{-2d}{3} \\ c = \frac{d}{3} \end{bmatrix}$$

Plane

$$-dx - \frac{2d}{3}y + \frac{d}{3}z + d = 0$$

$$-x - \frac{2}{3}y + \frac{1}{3}z + 1 = 0$$

$$x + \frac{2}{3}y - \frac{1}{3}z - 1 = 0$$

$$3x + 2y - z - 3 = 0$$

for $\langle m, 1-m, 4\rangle$

$$3m + 2(1-m) - 4 - 3 = 0$$

$$3m + 2 - 2m - 7 = 0$$

$$m - 5 = 0$$

$$\therefore \quad m = 5$$

[3.2] Solution 2 (using cross product)

$$\overrightarrow{PQ} = \langle -3, 4, -4 \rangle - \langle 4, -2, 5 \rangle = \langle -7, 6, -9 \rangle$$

$$\overrightarrow{PR} = \langle 1, 2, 4 \rangle - \langle 4, -2, 5 \rangle = \langle -3, 4, -1 \rangle$$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -7 & 6 & -9 \\ -3 & 4 & -1 \end{vmatrix} = \langle 30, 20, -10 \rangle = \langle 3, 2, -1 \rangle$$

Plane

$$3(x+3) + 2(y-4) - (z+4) = 0$$

$$3x + 2y - z + 9 - 8 - 4 = 0$$

$$3x + 2y - z - 3 = 0$$

m in plane

$$3m + 2(1-m) - 4 - 3 = 0$$

$$3m - 2m + 2 - 4 - 3 = 0$$

$$m - 5 = 0$$

$$\therefore \quad m = 5$$

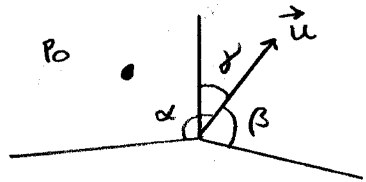
[4] $P_0(3,-1,2)$ on ℓ. Direction cosines are $\cos 60, \cos 45, \cos 60$.

$\ell \parallel \vec{u}$, $|\vec{u}| = 1$.

Then $u_1 = |\vec{u}| \cos 60 = \frac{1}{2}$

$u_2 = |\vec{u}| \cos 45 = \frac{\sqrt{2}}{2}$

$u_3 = |\vec{u}| \cos 60 = \frac{1}{2}$



ℓ: $\langle 3,-1,2\rangle + t \langle \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\rangle$

$$\begin{aligned} x &= 3 + \tfrac{1}{2}t \\ y &= -1 + \tfrac{\sqrt{2}}{2}t \\ z &= 2 + \tfrac{1}{2}t \end{aligned} \iff \begin{aligned} t &= 2(x-3) \\ t &= \frac{2(y+1)}{\sqrt{2}} = \sqrt{2}(y+1) \\ t &= 2(z-2) \end{aligned}$$

so ℓ: $2(x-3) = \sqrt{2}(y+1) = 2(z-2)$

[5] l_1 and l_2 each parallel to $\langle 2, 3, 1 \rangle$.

To find eqn plane, use eqns for l_1, l_2 to find 3 non co-linear points in plane.

l_1 $\quad x = 2t, \ y = 3t + 2, \ z = t - 4$

$t = 0 \Rightarrow P_1(0, 2, -4)$

$t = 1 \Rightarrow P_2(2, 5, -3)$

l_2 $\quad x = 2t + 1, \ y = 3t, \ z = t$

$t = 0 \Rightarrow P_3(1, 0, 0).$

Since P_1, P_2, P_3 in plane,

$$\begin{bmatrix} 2x + 5y - 3z + d = 0 \\ 2y - 4z + d = 0 \\ x + d = 0 \end{bmatrix} \equiv \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

$\equiv \quad d = t, \ c = \frac{t}{2}, \ b = \frac{t}{2}, \ a = -t$

$-tx + \frac{t}{2}y + \frac{t}{2}z - 1 = 0$ $\quad$ EQN Plane

$\therefore \quad 2x - y - z - 2 = 0$ is EQN of Plane

[6] $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z+2}{5}$, $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z+2}{3}$

$x = 3t+1$ $\quad$ $x = 2t+1$

$y = 4t+3$ $\quad$ $y = 5t+3$

$z = 5t-2$ $\quad$ $z = 3t-2$

$3t+1 = 2t+1 \Rightarrow t = 0$

$4t+3 = 5t+3 \Rightarrow t = 0$

$5t-2 = 3t-2 \Rightarrow t = 0$

So ℓ_1 intersects ℓ_2 at $(x, y, z) = (1, 3, -2)$

$\ell_1 \parallel \langle 3, 4, 5 \rangle$, $\ell_2 \parallel \langle 2, 5, 3 \rangle$, $\vec{n} = \langle a, b, c \rangle$ = normal to plane

$\vec{n} = \langle 3, 4, 5 \rangle \times \langle 2, 5, 3 \rangle = \langle -13, 1, 7 \rangle$

SO EQN PLANE, USIN $P(1, 3, -2)$, $\vec{n} = \langle -13, 1, 7 \rangle$ is,

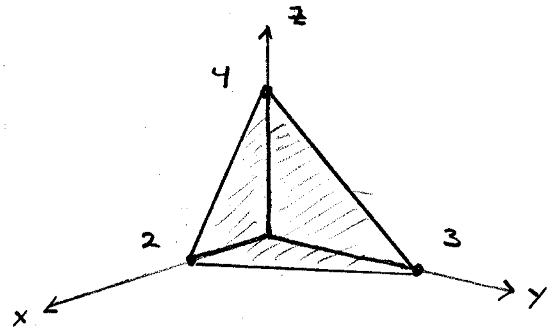
$-13(x-1) + (y-3) + 7(z+2) = 0$

$-13x + y + 7z + 13 - 3 + 14 = 0$

$-13x + y + 7z + 24 = 0$

$\therefore \quad 13x - y - 7z - 24 = 0$

[7]



$V_{pyramid} = \frac{1}{3} B h$

Take base as the triangular region in xy-plane

$A_{\Delta} = \frac{1}{2}(2)(3) = 3$ sq units

$h = 4$ units

$\therefore \quad V = \frac{1}{3}(3)(4) = 4 \text{ units}^3$

[8] $\alpha : \quad 4x + 3y + 5z = 50$

[8.1] d $O(0,0,0)$ to α.

$$d = \frac{|0+0+0-50|}{\sqrt{16+9+25}}$$

$$= \frac{50}{5\sqrt{2}}$$

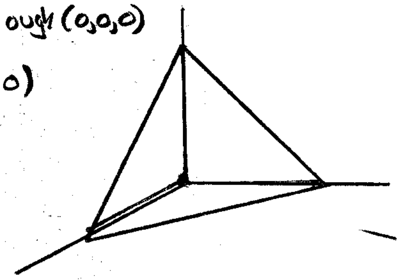
$$= \frac{10}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}}{2}$$

$$\therefore \quad \boxed{d = 5\sqrt{2}}$$

[8.2] Point $P(x, y, z)$ sym to $(0,0,0)$ w.r.t α.

P on line perpendicular to α through $(0,0,0)$
a distance of $2 \cdot 5\sqrt{2}$ from $(0,0,0)$



$$5\sqrt{2} = \frac{4x + 3y + 5z}{\sqrt{16+9+25}}$$

P on $\langle 0,0,0 \rangle + t \langle 4,3,5 \rangle$

$$x = 4t, \quad y = 3t, \quad z = 5t$$

Then,

$$\frac{4(4t) + 3(3t) + 5(5t)}{5\sqrt{2}} = 5\sqrt{2}\,(2)$$

$$16t + 9t + 25t = 25 \cdot 2 \cdot 2$$

$$50t = 100$$

$$t = 2$$

when $t = 2$

$$x = 8, \quad y = 6, \quad z = 10$$

$\therefore$ $P(8, 6, 10)$